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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The final results and academic achievements are reported on a program aimed at investigation of external burning propulsion. Concentration was aimed at bluff-base, axisymmetric bodies in supersonic flight with an approach boundary layer that is fully turbulent. An adiabatic near wake theory was developed which is in excellent agreement with experiment. A parametric investigation of external burning performance showed that while high thrust levels can be achieved the specific impulse is lower than expected. Consequently, future studies should concentrate upon a combination of base burning and external burning.

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Analytical Investigation of External Burning behind Bluff-Base Axisymmetric Bodies

by

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Final Report

U. S. Army Research Office

Grant Nos. DAHC04-73-C-0038

and DAAG29-77-G-0023

October 10, 1977

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Abstract

The final results and academic achievements are reported on a program aimed at investigation of external burning propulsion. Concentration was aimed at bluff-base, axisymmetric bodies in supersonic flight with an approach boundary layer that is fully turbulent. An adiabatic near wake theory was developed which is in excellent agreement with experiment. A parametric investigation of external burning performance showed that while high thrust levels can be achieved the specific impulse is lower than expected. Consequently, future studies should concentrate upon a combination of base burning and external burning.

Table of Contents

Introduction

There are a great number of potential air-to-air, air-to-ground and ground-to-air weaponry missions that require either a sustain or mild acceleration phase of the missile trajectory or which could benefit from a substantial drag reduction during a portion of the trajectory. Furthermore, many of these missions require operation sufficiently low in the atmosphere that airbreathing propulsion, if it is competitive with the rocket is attractive.

Investigated in this program was the concept of external burning. In its purest form this consists of burning outside of all shear layers near the body, in the adjacent supersonic stream. Moreover, this program concentrated upon external burning behind axisymmetric bodies with a turbulent approach boundary layer. The idea is that compression waves focusing upon the near wake will be able to transmit the high pressure through the subsonic portion of the wake and, hence, raise the base pressure. This concept is not to be confused with base burning, whereby a combustible is entrained into the near wake. While base burning is also promising for base pressure elevation, the concentration on external burning was motivated by the desire for high base thrust levels.

The program was analytical in nature but used data from other programs to validate the theory as it was developed. The program proceeded in two steps. The first was to create an accurate, but computationally fast, theory for the near wake. The second was to incorporate external burning into the theory. The results of the first effort are attached as Appendix A and of the second as Appendix B.

Summary of Results

An integral method was used to construct a theory of the near wake behind bluff base bodies in supersonic flight with a turbulent approach boundary layer. The method is computationally fast and has been checked against numerous experiments, with excellent agreement in base pressure, wake length scales, and detailed field quantities. The following conclusions may be drawn from the theory:

(1) The solution of the inviscid supersonic flow adjacent to the near wake plays a strong role in the overall solution. It must be treated in a nearly exact (method of characteristics) manner.

(2) The base pressure is a strong function of flight Mach number and decreases with a Mach number increase. However, detailed flow field quantities such as the location of the rear stagnation point and velocity on the dividing streamline are only weak functions of Mach number.

(3) The base pressure is a weak function of upstream boundary layer thickness. However, the detailed flow field quantities are strongly affected.

(4) One area where there is poor agreement between theory and experiment is that of base bleed. The theory underpredicts the value of base pressure for a given bleed rate.

(5) The integral method, which used the boundary layer approximation for the viscous near wake, ran into some 'singularity' problems with some velocity profiles and governing equations. More investigation is warranted to determine the nature of these singularities, and to systematize the procedure for overcoming these. Experiments show that,

in fact, the boundary layer approximation is poor near the base plane; this assumption should be relaxed in future treatment of the problem.

The above wake theory was then combined with a theory incorporating combustion external to the near wake. The combustion zone was treated as a nearly one-dimensional premixed region with a centrifugal correction. The following conclusions may be drawn.

(1) The base pressure may be elevated to a net thrust condition by this method, but the specific impulse performance with state-of-the-art fuel rich propellants is poor; it may be beaten by a conventional rocket. Consequently, it is believed that the pure external burning concept should not be considered alone, but in conjunction with base burning. External burning provides the means for high thrust, whereas base burning can provide high efficiency at lower base pressure rises.

(2) The primary variables that affect external burning performance are Mach number, fuel-air ratio, total combustible mass flow and the fuel calorific value. The performance is insensitive, within reasonable tolerance, to heating zone placement and heat release distribution.

(3) It is imperative that a theory be constructed for a combined base burning and external burning system in order that an upper limit of performance can be established.

Summary of Publications

The following publications were generated in this program:

1. Strahle, W. C. and Mehta, G. K., "Turbulent Axisymmetric Base Flow Studies for External Burning Propulsion," CPIA Publication 261, Vol. 2, p. 441 (1974).

2. Strahle, W. C., Mehta, G. K. and Hubbartt, J. E., "Progress on a Base Flow Model for External Burning Propulsion," Aerodynamics of Base Combustion (Murthy, ed.), AIAA, New York (1975) p. 339.

3. Strahle, W. C., Mehta, G. K., Hubbartt, J. E., Neale, D. H. and Pronchick, S. W., "Turbulent Axisymmetric Base Flow Studies for External Burning Propulsion," CPIA Publication 273, Vol. 2, p. 389 (1975).

4. Strahle, W. C., Hubbartt, J. E., Neale, D. H., Mehta, G. K. and Wilson, W. W., "Turbulent Axisymmetric Base Flow Studies for External Burning Propulsion," CPIA Publication 281, Vol. 3, p. 233 (1976).

5. Mehta, G. K. and Strahle, W. C., "A Theory of the Supersonic Turbulent Axisymmetric Near Wake behind Bluff-Base Bodies, AIAA Journal, 15, p. 1059 (1977).

Summary of Academic Achievements

Dr. Gopal K. Mehta obtained his Ph.D degree under sponsorship of this program.

Appendix A

A Theory of the Supersonic Turbulent Axisymmetric Near Wake behind Bluff-Base Bodies

A Theory of the Supersonic Turbulent Axisymmetric Near Wake behind Bluff-Base Bodies

Gopal K. Mehta* and Warren C. Strahle†
Georgia Institute of Technology, Atlanta, Ga.

Theme

ONE of the current interests in solving the near wake problem in a supersonic flow is to predict the effect of various base drag reducing concepts. The external burning concept, as applied by Strahle[1] to the two-dimensional case using the simple Crocco-Lees[2] theory showed that significant base drag reduction can be obtained using this concept. The motivation of the present investigation was to apply this concept to a more practical projectile shape, viz., axisymmetric, in a more quantitatively exact way. This now is warranted because of the notable improvement in the Crocco-Lees approach by Reeves and Lees[3] for the laminar case and the remarkably successful application of the Reeves and Lees analysis by Alber and Lees[4] to the two-dimensional turbulent case, using an eddy viscosity concept. Here, Alber and Lees' work, with suitable modifications, has been extended to the axisymmetric case. The present theory is for the adiabatic axisymmetric supersonic turbulent near wake without combustion, but contains sufficient flexibility to incorporate simple external combustion. The reversed flow has been treated with sufficient detail so as to allow for base bleed, because reactive bleed may be a promising adjunct to external burning to improve performance. Also, an entropy layer in the inviscid stream is allowed for, because several concepts of external burning would introduce the fuel by injection into the supersonic stream, causing injection shocks.

Contents

Complete details of the present work and the references are given in Ref. 5. The flow model used for the present analysis is as shown in Fig. 1. The lip shock and the lip-shock/wake-shock interaction have been neglected, since they have been shown by previous studies to be of secondary importance at moderate Mach numbers.

The analysis essentially consists of solving the corner flow region and the flow downstream of the base. The solution of the corner region provides the initial conditions for the wake analysis. Earlier work showed that after such extreme expansions as occurring in the present case, viscous forces remain predominant only in a small part Δ_2 of the expanded boundary layer, corresponding to the portion Δ_1 of the initial boundary layer δ_1. These thicknesses are found by assuming a 1/7th power profile for the initial boundary layer, a value for the base pressure, $(\partial u/\partial r)_{\Delta_2} = (\partial u/\partial r)_{\Delta_1}$, and an isentropic expansion in the outer streamtubes, for given upstream Mach number M_1, pressure p_1, momentum thickness θ_1, and base radius R.

The flow downstream of the base, as shown in Fig. 1, consists of four regions. The inner regime, consisting of the recirculatory region [1] and shear flow region [2], is represented by integrated boundary-layer equations. The turbulent shear stress is evaluated by following an approach similar to that of Alber and Lees. An eddy viscosity model is chosen differently for the region upstream of the RSP and downstream of the RSP. The constant, and the characteristic length and velocity scales in the incompressible eddy viscosity models are defined to recover the well-known results of the self-preserving shear layer and the far wake, and to give a continuous variation of the viscosity at the RSP. This incompressible eddy viscosity then is multiplied by a compressibility factor to obtain the proper Mach number dependence.

The solution from the base is started using the Green[6] two-parameter incompressible velocity profiles, which contain a constant velocity core. The use of these profiles results in six unknowns in the problem, for an assumed value of the base pressure. The mass conservation equation, the momentum conservation equation, the mechanical energy equation, and the centerline momentum equation, in conjunction with the solution of the outer regime, are used to solve for these unknowns. The outer regime consists of the inviscid rotational region [3] and the inviscid isentropic region [4]. Consistent with the accuracy and the spirit of the present approach, this regime is solved by an approximate method of characteristics (AMC) by modifying Webb's[7] method.

As one moves downstream of the base, the initial constant velocity core is eaten up by the shear layer, and the number of parameters is reduced by one. Better matching of the maximum reverse velocity with the experiments takes place if a small constant velocity core in the velocity profiles is carried through downstream, or the Kubota et al.[8] one-parameter profiles are allowed to take over. However, it is not always possible to match the starting Green's profiles with the Kubota et al. profiles. Anyway, the unknowns are reduced by one at a unique matching point. The solution of the three integrated equations and the outer regime is sufficient to calculate the five unknowns. The solution is continued downstream until the Crocco-Lees singularity is hit. If the initial value of base pressure guessed is low, the determinant of the solution matrix goes through zero before any of the numerators and vice versa. It requires from 3 to 10 sec to complete one iteration on a CDC CYBER 70-74.

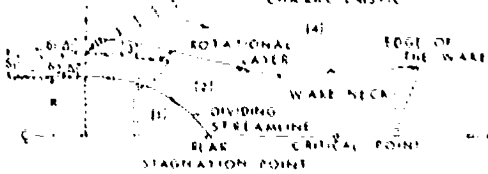

Fig. 1 Axisymmetric supersonic near wake model; δ_2 is thickness corresponding to Δ_2 after separation.

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Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Supersonic and Hypersonic Flow; Viscous Nonboundary Layer Flows.

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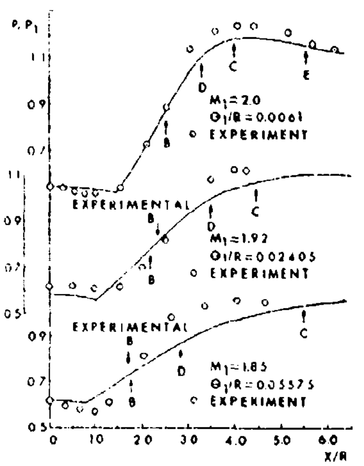

Fig. 2 Centerline pressure variation with axial distance and comparison with experiments near Mach 2; B = rear stagnation point, C = Crocco-Lees critical point, D = wake neck, E = centerline Mach 1 point.

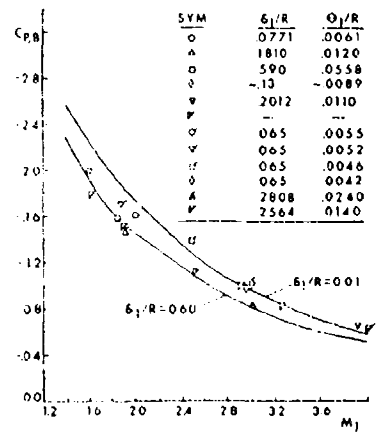

Fig. 3 Effects of Mach number on base pressure coefficient.

Figures 2 and 3 show a comparison of some of the results of the present theory with the experimental data, and, as seen, good agreement is obtained. However, theory seems to slightly overpredict the base pressure at large Mach numbers (~ above 2.5) and slightly underpredict the base pressure at low Mach numbers (~ below 1.8). It is found that, if the eddy viscosity is multiplied by a factor $\sqrt{M_1}/2$, a much better correlation with the experimental results can be obtained for a wider range of Mach number. Also, the variation of centerline Mach number and the effect of boundary-layer thickness on base pressure compared well with experiments. These comparisons with the experiments thus show the adequacy of the present modeling of the corner region and the shear stress. A number of studies, using the present program, were made and the major conclusions arrived at are as follows:

1) The solution of outer regime plays a very important role in the present problem; the replacement of AMC with the Prandtl-Meyer relation resulted in very poor results.

2) For better prediction of flowfield details, a good velocity profile is essential. However, base pressure is affected in a minor way because of a change in profiles.

3) The base pressure is a strong function of upstream Mach number and decreases with an increase of Mach number. The detailed quantities, such as location of the RSP, velocity on dividing streamline, etc., are only weak functions of Mach number.

4) The base pressure is a weak function of the upstream boundary-layer thickness. All other quantities such as centerline Mach number, etc. are strong functions of δ_1. Hence, measurement of the latter quantities can shed more light on and help in improving the corner model.

5) At low base bleed rates, the theory shows a much smaller base pressure rise compared with experiments. The inaccuracy of the boundary-layer equation, in representing the region close to the base, may be the major reason for this result.

In conclusion, the method developed here is computationally fast, provides adequate details of the near wake, and hence should be quite useful for preliminary design purposes.

Acknowledgment

This work was supported by the Army Research Office under Contract number DAHCOA-73-C-0038. Useful discussions with J. E. Hubbartt are acknowledged gratefully.

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Appendix B

Analysis of Axially Symmetric External Burning Propulsion for Bluff-Base Bodies

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Abstract

A parametric study for an external burning propulsion system was carried out to investigate potential performance and application of such a system. The study used a simplified analysis for an external burning zone incorporated in a realistic near wake axisymmetric supersonic turbulent base flow model based on the Crocco and Lees approach. The axisymmetric base flow model without combustion has previously been shown to agree well with experiment. The combustion zone is treated as a quasi one-dimensional zone, and is considered both shear free and non-conducting. With these major assumptions, it is shown that it is possible to achieve very high base pressures using external burning. The specific impulse values obtained are lower than reported for the two-dimensional case. In many cases the performance is less than usually achievable by conventional rocket propulsion, if current fuel rich gas generators provide the fuel for the ex-

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ternal burning propulsion system. Recent experiments carried out using compression surfaces to simulate external burning confirm the order of magnitude of specific impulse obtained by the theory.

Nomenclature

A	centerline velocity parameter in Green's profiles; area
A_u	$\int_0^\delta \frac{u}{u_e}\, rdr$
A_1	$\int_0^\delta \frac{\rho u}{\rho_e u_e}\, rdr$
A_2	$\int_0^\delta \frac{\rho u^2}{\rho_e u_e^2}\, rdr$
A_3	$\int_0^\delta \frac{\rho u^3}{\rho_e u_e^3}\, rdr$
c	speed of sound
c_p	specific heat
F	fuel-air ratio
g_o	acceleration [illegible] gravity
h	static enth[illegible] height of core region in Green's profiles
H	stagnatio[illegible]
H_f	h[illegible] per unit mass of fuel
I	injection parameter , $\frac{\text{mass injected}}{\rho_{e1}\, u_{e1}\, A_b}$
I_{sp}	specific impulse
K_1, K_2	constants
ℓ	shear layer thickness
$\dot{m}_{1-D}$	mass flow rate in one-dimensional zone
M	Mach number
n	natural normal coordina[illegible]

p	pressure
r	radial coordinate
r_H	upstream radial location of upper streamline of the one-dimensional zone
r_L	upstream radial location of the lower streamline of one-dimensional zone
R	gas constant; base radius
R_1	$- \frac{1}{\rho_e u_e^3} \int_0^\delta u \frac{\partial}{\partial r} (r \tau_t) dr$
S	entropy
T	temperature
u	axial velocity
v	radial velocity
V	total velocity
x	axial coordinate
x_E	axial position of the termination of heat addition
x_H	axial position of the starting of heat addition
α	Mach angle
γ	ratio of specific heats
Δ_1	thickness of inner portion of boundary layer
Δ_2	thickness of inner portion after expansion
δ	boundary layer and wake thickness
δ_2	thickness of viscous layer after separation
η	incompressible radial coordinate ; characteristic direction
θ	flow angle
ν	Prandtl-Meyer angle
ξ	characteristic direction
ρ	density

τ_t turbulent shear stress

τ_1, τ_2 corrections due to non-zero velocity gradients caused by entropy gradients at the edge of the wake

χ heat distribution parameter

Subscripts

b value at the base

e value in external flow

i incompressible quantities

o value at centerline ; stagnation quantity

∞ value at large distance

1 upstream condition

Superscript

$-$ average quantities in one-dimensional zone

1. Introduction

There are many flight missions which require or may benefit from substantial drag reduction or elimination during a portion of a supersonic trajectory. It has been recognized for some time that external burning during supersonic flight can produce compression waves which can be translated into propulsive forces on a flight vehicle. This external burning concept is different from the relatively better explored base burning concept(1). In the external burning method, the combustion mostly takes place in the inviscid flow adjacent to the viscous wake, while in the base burning method, the low velocity combustible mixture, injected through the base, burns in the viscous wake. There is a limitation on the maximum base pressure rise achievable in the base burning concept. There is no such limitation in the external burning concept. The external burning method has also been claimed to be simple in design and to give

high performance.

The external burning concept has initially been investigated in the two dimensional case by Strahle,[2] using the original Crocco and Lees theory[3] of the near wake flow. He showed, (i) base pressures much higher than the ambient pressure, and (ii) reasonable specific impulse values can be achieved using this concept. Smithey[4] and recently Neale et al.[5], carried out experiments with axisymmetric bodies, using compression surfaces to simulate external burning, at Mach 2 and 3, respectively. Their results confirmed the first finding of high attainable base pressure. But much smaller specific impulses were indicated than anticipated. Smithey[4] also applied the external burning concept to the axisymmetric case, using, again, the original Crocco and Lees theory. This original theory is known to contain an inadequate prediction of the length scales of the near wake. Recently, the more detailed theory of Alber and Lees[6], applicable to the two-dimensional turbulent case, was extended by the present authors[7] (and also independently by Peters et al.[8]) to the axisymmetric case; it was shown to accurately predict the near wake details. In the present paper, this theory has been modified to treat external burning propulsion. Calculations are carried out to predict the performance of such a system more realistically.

A short outline of the theory without combustion and the basic flow model used in this theory are presented in Section II. For more details, one is referred to the back up paper of Reference (7) or to Reference (9).

Section III describes the details of the heat addition zone added in the inviscid flow region of the above model. Fuel is considered fully dispersed in this zone and there is no mixing across bounding streamlines. The entrainment of the air before combustion can be taken into account by shifting the initial position of the streamlines. In reality, there will be a diffusion flame; this

model may be thought of as an effective one dimensional smoothing of the effects of a diffusion flame. On either side of the combustion zone the flow field is treated by the method of characteristics (MOC). Between the near wake boundary and the heat addition zone the MOC is rotational to allow for injection shock entropy rise and to treat the shedding of the initial boundary layer. Outside the heat addition zone the MOC is irrotational. The heat addition zone is displaced outward a sufficient amount so that heated streamlines do not intersect the near wake boundary before the wake closure singularity (Crocco-Lees critical point) is encountered in a downstream numerical integration. This allows the near wake to be treated under an adiabatic assumption. Thus, the analysis of the inner region and the corner region remain unchanged, except for slight modifications in the program to take into account the possible compression instead of expansion at the corner.

In the Appendix some computations, using the Mach number distribution provided by the compression surfaces of the experimental set up of Reference (5), are shown. The agreement with experiment is good, although the base pressures obtained are about 15% less than those obtained in the experiments.

Section IV deals with the results and discussion of the present external burning near wake theory. It is first shown that it is possible to achieve high base pressure rises without failure of the present computational scheme. However, the total temperature rise and other values of the parameters have been limited either by transonic Mach numbers in the heat addition zone or by the heat addition zone mixing with the viscous wake. Next, a parametric study, based upon the parameters obtained through dimensional analysis, is carried out. The reference conditions chosen for carrying out this study, although arbitrary, are selected, firstly, to give a large variation of

various parameters before any of the above mentioned limitations come into the picture. Secondly, they are chosen to avoid large gradients in field quantities; large gradients require finer step size and hence more computer time. With this study, the importance of the various parameters are delineated.

Other than the upstream Mach number and the initial boundary layer thickness, which are more or less design inputs, it is essential to have an input as to how to distribute the combustion to obtain the most efficient utilization, and this also has been discussed in a general manner in this section.

II. Highlights of the Theory of the Near Wake Without Combustion

This theory, which has been presented in Reference (7), is an extension of the work of Alber and Lees[6] on two-dimensional turbulent supersonic near wakes. The major points of variance are the treatment of the corner region, the detailed form of the eddy viscosity model, and, of course, the treatment of the external flow. The model used to represent the near wake flow field is as shown in Fig. 1. A uniform supersonic flow at constant pressure and Mach number, with a finite boundary layer thickness, approaches the base of an axisymmetric body. It undergoes an expansion near the corner. The viscous part beyond the base consists of (1) the recirculatory region and (2) the shear layer region, while the inviscid part consists of (3) the rotational layer, which merges into the shear layer, and (4) the irrotational region.

The flow diagram of the approach is shown in Fig. 2. The input variables are M_{e1}, δ_1/R and I. The unknown of the problem is base pressure, and to start with, this is assumed. The complete flow field is assumed to be adiabatic with turbulent Prandtl number equal to 1 and to obey the perfect gas laws. For the sake of analysis, the flow field is divided into two parts, viz., the corner

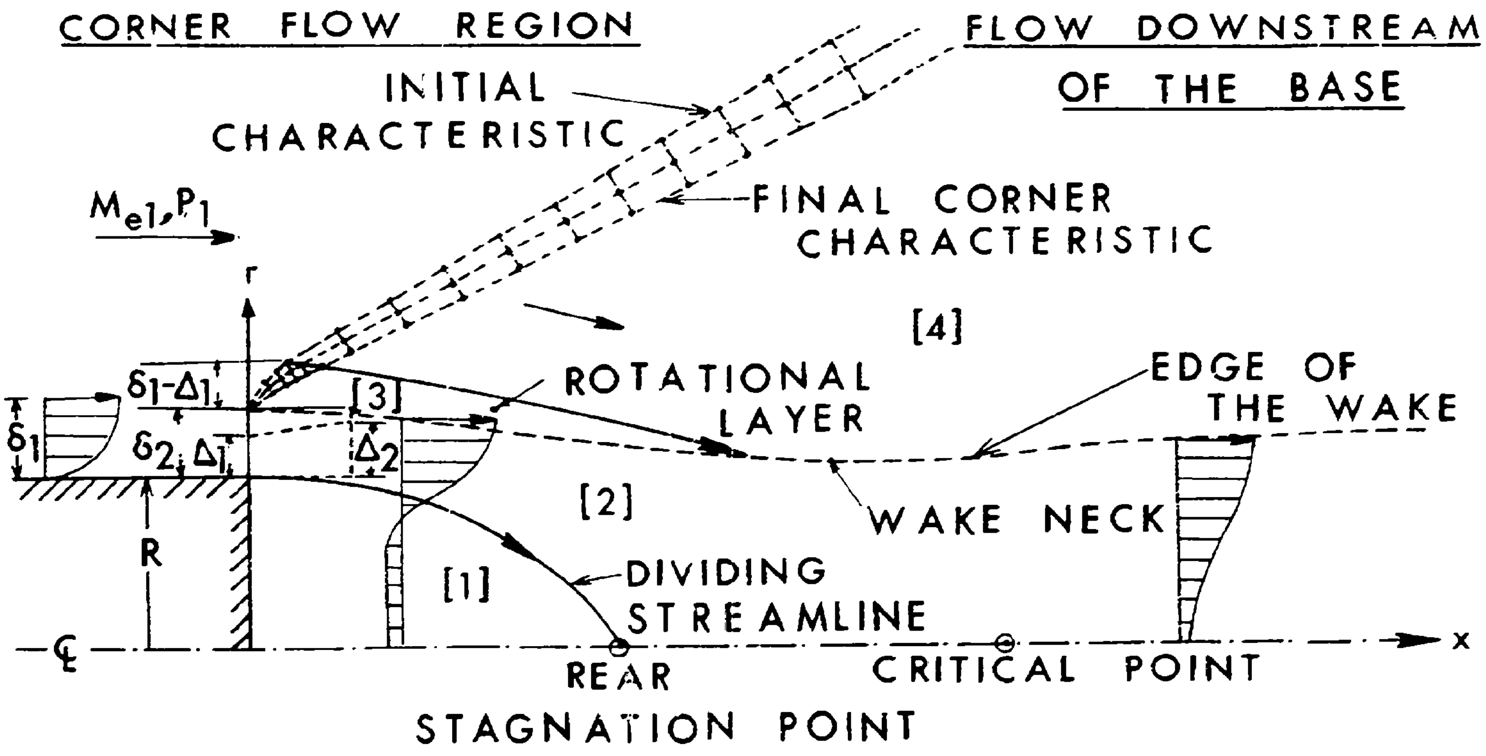


Figure 1. Axisymmetric Near Wake Model.

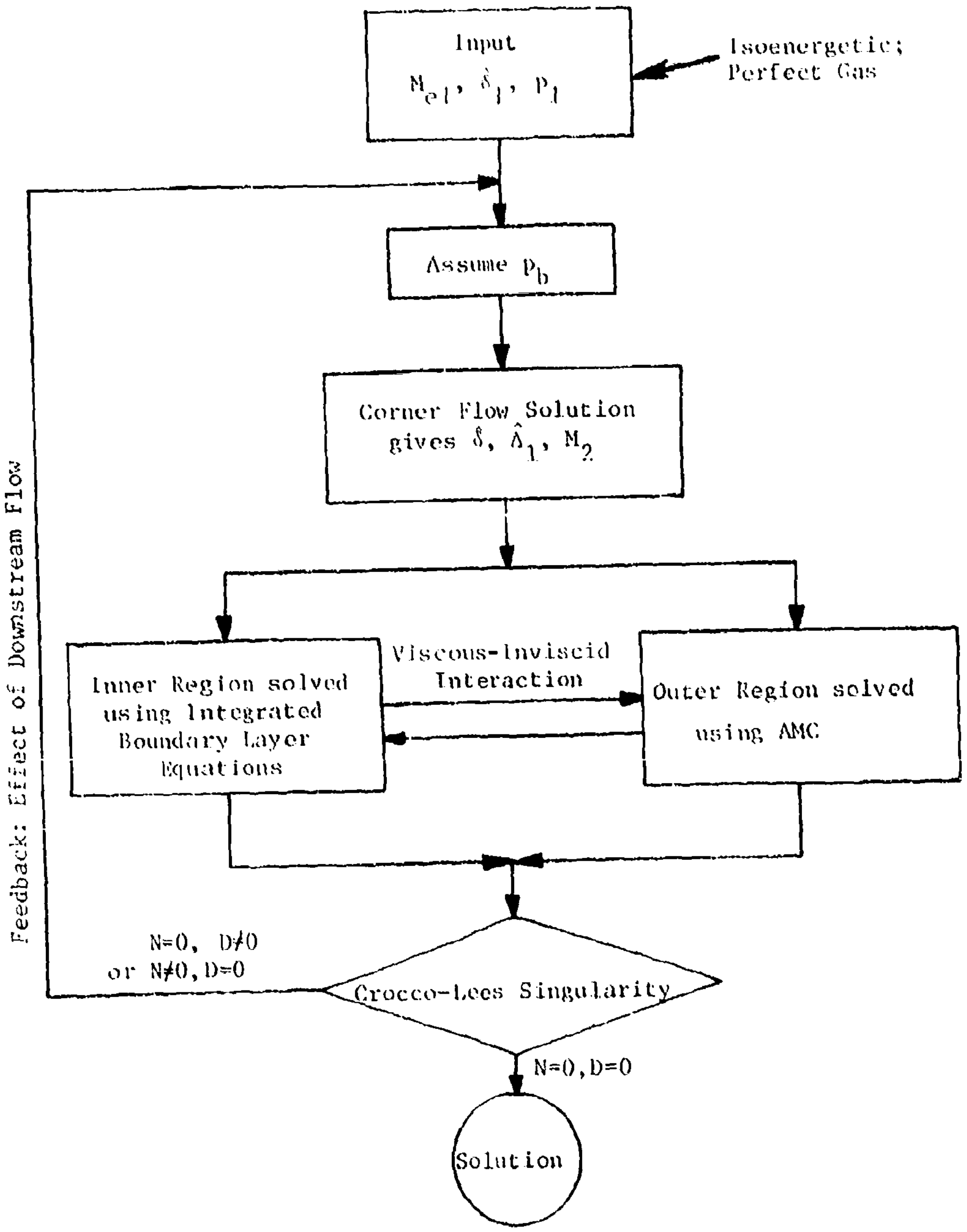


Figure 2. Flow Diagram of the Approach.

flow region and the flow downstream of the base. The solution of the corner region provides the initial conditions for the flow downstream of the base. Specifically, the thickness of the boundary layer in which the viscous forces remain predominant is determined by this solution. This region is solved by an approximate model. The major stipulations in this model are that the initial boundary layer can be represented by a 1/7th power profile, $\left.\frac{\partial u}{\partial r}\right)_{\Delta_2} = \left.\frac{\partial u}{\partial r}\right)_{\delta_1}$, and there is an isentropic expansion in the outer streamtubes. Downstream of the base, the outer inviscid regime, consisting of regions (3) and (4), is solved using an approximate method of characteristics (AMC). This method is a modification of Webb's[10] method, and is consistent with the accuracy and spirit of this approach. The inner viscous regime, consisting of region (1) and (2), is represented by integral boundary layer equations. The governing equations used to represent this region in the present formulation are given below.

Mass conservation

$$\frac{d}{dx}\left(\rho_e u_e A_1\right) - \rho_e u_e \delta \frac{d\delta}{dx} = - \rho_e v_e \delta \qquad (1)$$

Momentum conservation

$$\frac{d}{dx}\left(\rho_e u_e^2 A_2\right) - u_e \frac{d}{dx}\left(\rho_e u_e A_1\right) = - \frac{\partial p}{\partial x}\frac{\delta^2}{2} + \tau_1 \qquad (2)$$

Mechanical energy conservation

$$\frac{d}{dx}\left(\frac{\rho_e u_e^3 A_3}{2}\right) - \frac{u_e^2}{2}\frac{d}{dx}\left(\rho_e u_e A_1\right) = - \frac{\partial p}{\partial x} u_e A_u - \rho_e u_e^3 R_1 + \tau_2 \qquad (3)$$

In addition to these, one more equation is needed near the base for the chosen velocity profiles. This is taken, for simplicity, as the centerline momentum equation.

$$\frac{dp}{dx} = - \rho_o u_o \frac{du_o}{dx} \qquad (4)$$

As the flow is turbulent, some modelling of the shear stress is required. Here shear stress is evaluated using the turbulent viscosity concept, the details of which are given in Reference (7). The integrals in the above equations are obtained in closed form using a compressible-incompressible transformation. The solution is started from the base using two parameter Green's profiles[11] which give a good representation of the velocity profile near the base, with or without uniform base bleed. These are given by

$$\begin{aligned} \frac{u}{u_e} &= 1 - 2A & 0 \le \eta \le h_i \\ &= 1 - A - A \cos \pi \left(\frac{\eta - h_i}{\ell_i}\right) & h_i \le \eta \le \delta_i \end{aligned} \qquad (5)$$

The use of these profiles results in six unknowns, viz., p (or u_e), A, v_e, δ, h_i/δ_i and S_e. The external flow solution provides v_e(or θ_e) and S_e, while the solution of Eqs. (1) - (4) provides the remaining unknowns. Away from the base, the velocity profiles are represented by the Kubota-Reeves-Buss[12] one parameter profiles. As the number of unknowns are reduced by one using these profiles, only Eqs. (1) - (3) are sufficient to represent the inner region. The uniqueness of the solution in this approach comes from the occurrence of the Crocco-Lees singular point downstream of the wake. The physical explanation of this singularity is that the near wake, which is subsonic in the mean, becomes supersonic in the mean after this point. Mathematically, this singularity imposes an extra constraint on the problem. And the base flow problem turns out to be a boundary value problem, with base

pressure as the eigenvalue, and a unique solution of the near wake is obtained.

The strong points of this theory are the small computer time required to solve the problem and accurate prediction of the near wake details.

III. Theory with External Burning

The type of external burning considered here is the concept which allows compression waves to impinge upon the near wake behind a bluff-base body and to thereby raise the base pressure. In order to gain a feel for the performance and to bring out the essential features of an external burning propulsive system, a simple model is used to represent the external combustion. The external burning is limited to the inviscid flow adjacent to the near wake as shown in Fig. 3. Fuel is considered fully dispersed in this zone and there is no mixing across boundary streamlines. The heat addition zone is displaced outward a sufficient amount so that heated streamlines do not intersect the near wake boundary before the wake closure singularity (Crocco-Lees critical point) is encountered in the downstream numerical integration. This allows the near wake to be treated under an adiabatic condition, just like in the non-burning case. On either side of the heat addition zone the flow field is treated by the AMC as in Section II. Between the near wake boundary and the heat addition zone the AMC is rotational to allow for injection shock entropy rise and to take care of the shedding of the initial boundary layer. The heat addition zone itself is treated by one-dimensional gasdynamics, but with a centrifugal correction. The governing equations used to describe this zone are the one-dimensional energy equation

$$\dot{m}_{1-D} \quad \Delta \bar{T}_o = \frac{\Delta H}{c_p} \qquad (6)$$

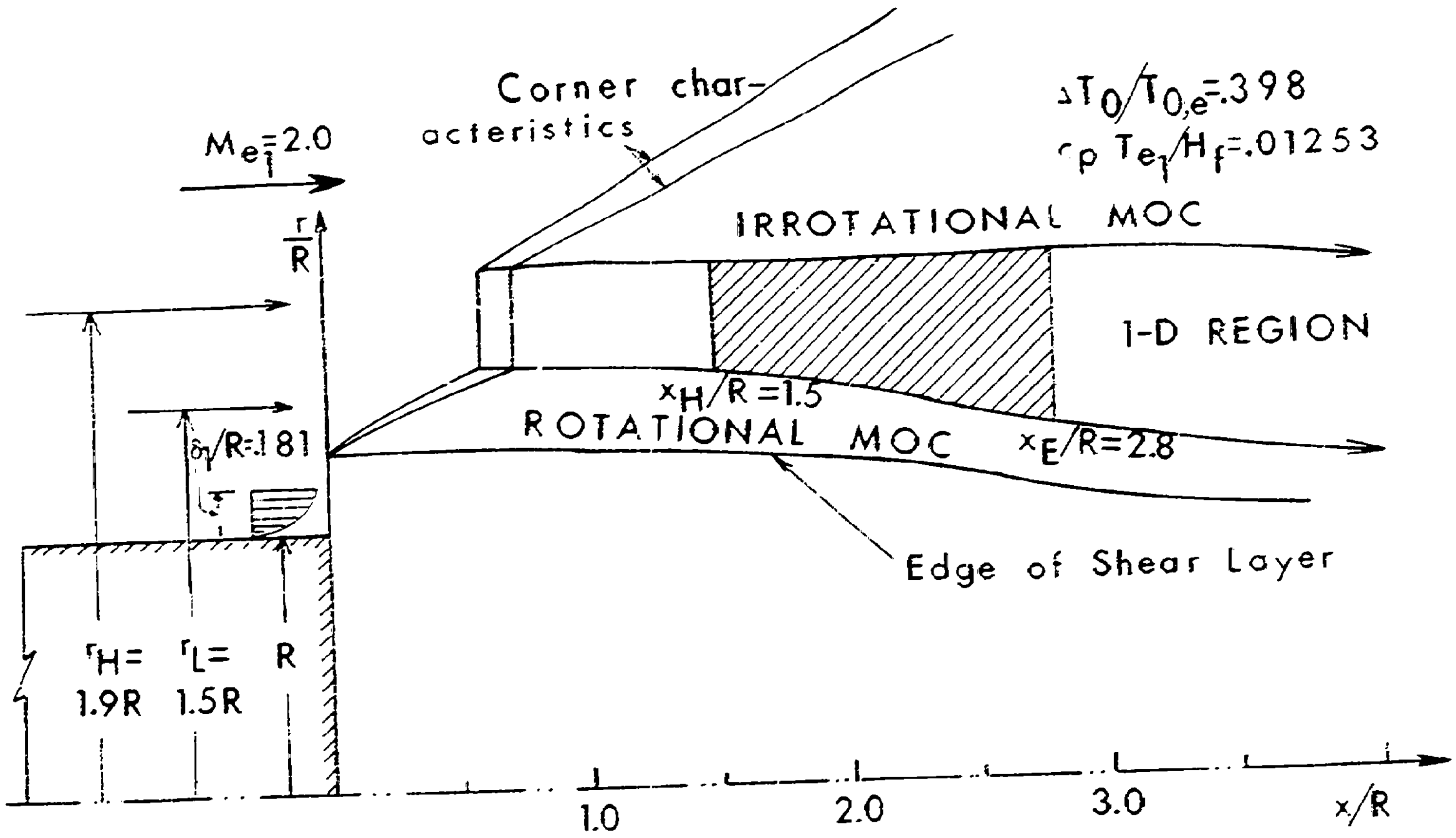


Figure 3. Axisymmetric Near Wake Flow Field with External Burning.

the one-dimensional continuity equation

$$\dot{m}_{1-D} = \bar{\rho}\,\bar{A}\,\bar{V} \tag{7}$$

the streamwise momentum equation

$$-\frac{d\bar{p}}{\bar{A}} = \dot{m}_{1-D}\,d\bar{u} \tag{8}$$

and the momentum equation normal to the streamlines

$$\bar{\rho}\,\bar{u}^2\,\frac{\partial\bar{\theta}}{\partial x} = -\frac{\partial\bar{p}}{\partial n} \tag{9}$$

Eq. (6), upon manipulation, can be rewritten as

$$\frac{1}{\bar{T}_o}\,\frac{d\bar{T}_o}{d\bar{x}} = \frac{F}{1+F}\left(\frac{T_{o,\infty}}{\bar{T}_o}\right)\frac{H_f}{c_p T_{o,\infty}}\,\frac{d\bar{h}}{d\bar{x}} \tag{10}$$

This gives the distribution of total temperature knowing the heat release rate function. However, in the present calculations, the distribution of total temperature is assumed; the overall energy equation is used to obtain the fuel-air ratio.

$$F = \frac{c_p\Delta\bar{T}_o/H_f}{1 - c_p\Delta\bar{T}_o/H_f} \tag{10a}$$

Eqs. (7), (8) and (9) upon differentiation and simplification give

$$\frac{1}{\bar{p}}\,\frac{d\bar{p}}{d\bar{x}} + \frac{1+(\gamma-1)\,\bar{M}^2}{1+\frac{\gamma-1}{2}\,\bar{M}^2}\,\frac{1}{\bar{M}}\,\frac{d\bar{M}}{d\bar{x}} + \frac{2\left(r_u\frac{dr_u}{d\bar{x}} - r_\ell\frac{dr_\ell}{d\bar{x}}\right)}{r_u^2 - r_\ell^2} - \frac{1}{2\bar{T}_o}\,\frac{d\bar{T}_o}{dx} = 0 \tag{11}$$

$$\frac{1}{\bar{p}}\,\frac{d\bar{p}}{d\bar{x}} + \frac{\gamma\bar{M}^2}{1+\frac{\gamma-1}{2}\,\bar{M}^2}\,\frac{1}{\bar{M}}\,\frac{d\bar{M}}{d\bar{x}} + \frac{\gamma}{2}\,\frac{\bar{M}^2}{\bar{T}_o}\,\frac{d\bar{T}_o}{d\bar{x}} = 0 \tag{12}$$

$$\frac{d\bar{\theta}}{d\bar{x}} = \frac{1}{\gamma \bar{M}^2} \frac{p_u - p_\ell}{\bar{p}} \frac{1}{r_u - r_\ell} \tag{13}$$

In addition to these, the geometrical relations needed are

$$\frac{r_u - r_{u-1}}{\Delta\bar{x}} = \tan \theta_u \tag{14}$$

$$\frac{r_\ell - r_{\ell-1}}{\Delta\bar{x}} = \tan \theta_\ell \tag{15}$$

Figure 4 shows the points referred to in the wake flow in the above equations and also the points referred to later. $\bar{p}$ and $\bar{\theta}$ are defined as

$$\bar{p} = \frac{p_u + p_\ell}{2}$$
$$\bar{\theta} = \frac{\theta_u + \theta_\ell}{2} \tag{16}$$

Also, as there cannot be a pressure difference or flow angle difference across a streamline, $p_\ell = p_{M12}$, $p_u = p_{1'}$, $\theta_\ell = \theta_{M12}$ and $\theta_u = \theta_{1'}$. However, velocity and temperature differences across streamlines are allowed; hence u_{M12} may not be equal to u_ℓ, etc.

The unknowns involved here are p_u, θ_u, M_u, p_ℓ, θ_ℓ, M_ℓ, r_ℓ, r_u and x_u. Eqs.(12)-(15), with the relations provided by the method of characteristics are sufficient to solve for these variables. The solution scheme used is a single variable (θ_1') iteration scheme, since it gives reasonable accuracy, and is explained below:

(i) Locate point M12 by extrapolation of ρu and θ, and using the mass conservation equation up to the rotational layer.

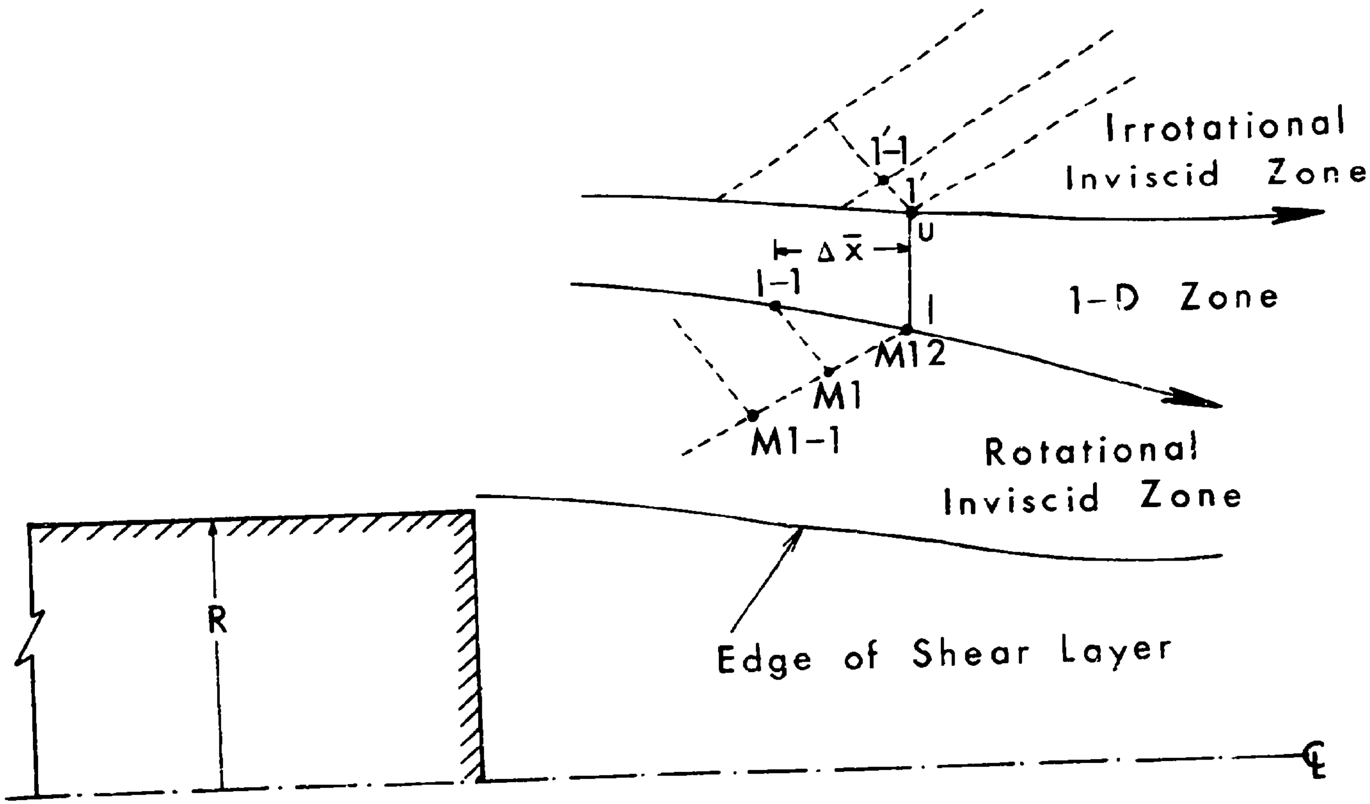


Figure 4. Some Geometrical Details.

(ii) Extrapolate ν_{M12} from the known values of ν at points M1 and M1-1. Calculate M_{M12}, and then p_{M12} using isentropic relations.

(iii) Obtain θ_{M12} using the relation valid for left running characteristics in the rotational flow, i.e.,

$$\theta_{M12} = \nu_{M12} - \nu_{M1} + \theta_{M1} - \frac{1}{M_{M1}} \frac{\sin \theta_{M1} \, \Delta \eta_{M12,M1}}{r_{M1}} + \frac{S_{M12} - S_{M1}}{\gamma R \tan \alpha_{M12} \, M_{M1}^{2}}$$

(iv) Obtain $d\bar{\theta}/d\bar{x}$ using Eq. (9), and the new $\bar{\theta}$. From Eq. (12), find θ_u. Also using geometrical relations (10) and (11), calculate r_u.

(v) Find $d\bar{M}/d\bar{x}$ using Eqs. (7) and (8), viz.

$$\frac{d\bar{M}}{d\bar{x}} = - \bar{M} \frac{1 + \frac{\gamma-1}{2} \bar{M}^2}{1 - \bar{M}^2} \left[\frac{\frac{dr_u^{\,2}}{dx} - \frac{dr_\ell^{\,2}}{dx}}{r_u^{\,2} - r_\ell^{\,2}} - \frac{1}{2\,\bar{T}_o} \left(1 + \gamma \bar{M}^2 \right) \frac{d\bar{T}_o}{d\bar{x}} \right] \tag{17}$$

Then obtain $d\bar{p}/d\bar{x}$ using Eq. (8), and hence, new values of $\bar{p}$ and p_1'.

(vi) Next obtain M_1' using the isentropic relation, and calculate ν_1'.

(vii) Obtain θ_1' using the relation along right running characteristics in the rotational region, i.e.,

$$\theta_{1'} = \nu_{1'-1} + \theta_{1'-1} - \nu_{1'} + \sin \mu_{1'-1} \frac{\sin \theta_{1'-1}}{r_{1'-1}} \Delta \xi_{1',1'-1}$$

(viii) If the above value of $\theta_{1'}$ is not the same as that of θ_u obtained in step (iv), guess a new value of p_{M12}, and obtaine the solution by iteration by repeating steps (iii) to (vii).

The specific impulse is obtained using its definition

$$I_{sp} = \frac{\text{Decrease in drag}}{\text{Mass flow rate of fuel}} = \frac{\Delta p_b\ A_b}{\dot{m}_f\ g_o}$$

which gives

$$I_{sp} = \frac{\Delta p_b/p_1}{F/(1+F)}\ \frac{p_1\ A_b}{\dot{m}_{1-D}\ g_o} \tag{18}$$

Finally, two major points of interest should be noted in the present analysis. First, consider Eq. (17). The second term in the parenthesis mostly dominates at least by one order of magnitude the contribution of the first term for large $\bar{M}$. But as more and more heat is added, as $\frac{d\bar{M}}{dx}$ is negative for $\bar{M} > 1$, $\bar{M}$ moves towards 1, and the order of magnitude of both the terms becomes equal. Thus the approximation of one dimensionality becomes very poor near $\bar{M}$ equal to 1. Secondly, in the concept of external burning, there is high loss of energy because very hot gases leave the system.

IV. Results and Discussion

The present calculations are made with a fuel of comparatively low calorific value, 10,000 Btu/lb, as this value is easily achievable from current fuel rich gas generators[13]. The freestream temperature is assumed to be 519°R. The combustion is assumed to be 100% efficient. The maximum ratio of fuel to air that is permitted is the stoichiometric fuel-air ratio, since no mixing or diffusion flame is allowed in the present model. However, a rough computation, using a simple fuel, showed that this limit is reached after the $\bar{M} = 1$ point, where the present computations are invalid. The molecular weight etc. of the mixture is taken as that of the air.

In order to check whether or not the inclusion of the one-dimensional zone would substantially degrade the original theory of the near wake, a

few no-heat addition cases with various dimensions of the one-dimensional zone were computed and compared with the original method using the AMC in the outer flow. The agreement between the centerline pressure distributions of the approximate and exact solutions is within 5% everywhere, showing that a one-dimensional treatment of the heat addition zone should not yield unacceptable error. However, this variation must be kept in mind when interpreting some of the results showing the effect of various parameters.

Figures 5 and 6 are the results of the computations carried out close to the $\bar{M} = 1$ limit, with total temperature rise (or fuel-air ratio, see Eq.(10-a)) as the parameter. These runs confirm the finding of previous research workers[2,4,5] that high base pressure rise is possible using this concept. Also, this figure gives a feel for the specific impulse values that can be expected at practical base pressure rises with this method. The specific impulse decreases, i.e., the system becomes less efficient, both with increase of the Mach number and increase of the fuel-air ratio. The minimum value of the specific impulse shown in Figure 5 is about 86 secs. occurring at Mach 3.0 with $\frac{\Delta\bar{T}_o}{T_{o,\infty}} = 2.27$, with a corresponding base pressure value equal to 1.85 times the ambient pressure. The maximum value of I_{sp} is about 198 secs. occurring at Mach 2.0 with $\frac{\Delta\bar{T}_o}{T_{o,\infty}} = 0.267$, with a corresponding base pressure value equal to 0.739 times the ambient pressure. The rear stagnation pressure and the base pressure rise with the increase of fuel-air ratio, but there is not a one-to-one correspondence in their rises. For a fixed Mach number, this is mainly due to the movement of the rear stagnation point (RSP). The location of the RSP tends to reach a limiting value at higher fuel-air ratio, and then $\Delta p_{RSP}/\Delta p_b$ becomes almost constant. This ratio is also affected by Mach number, and other parameters which move the location of the RSP.

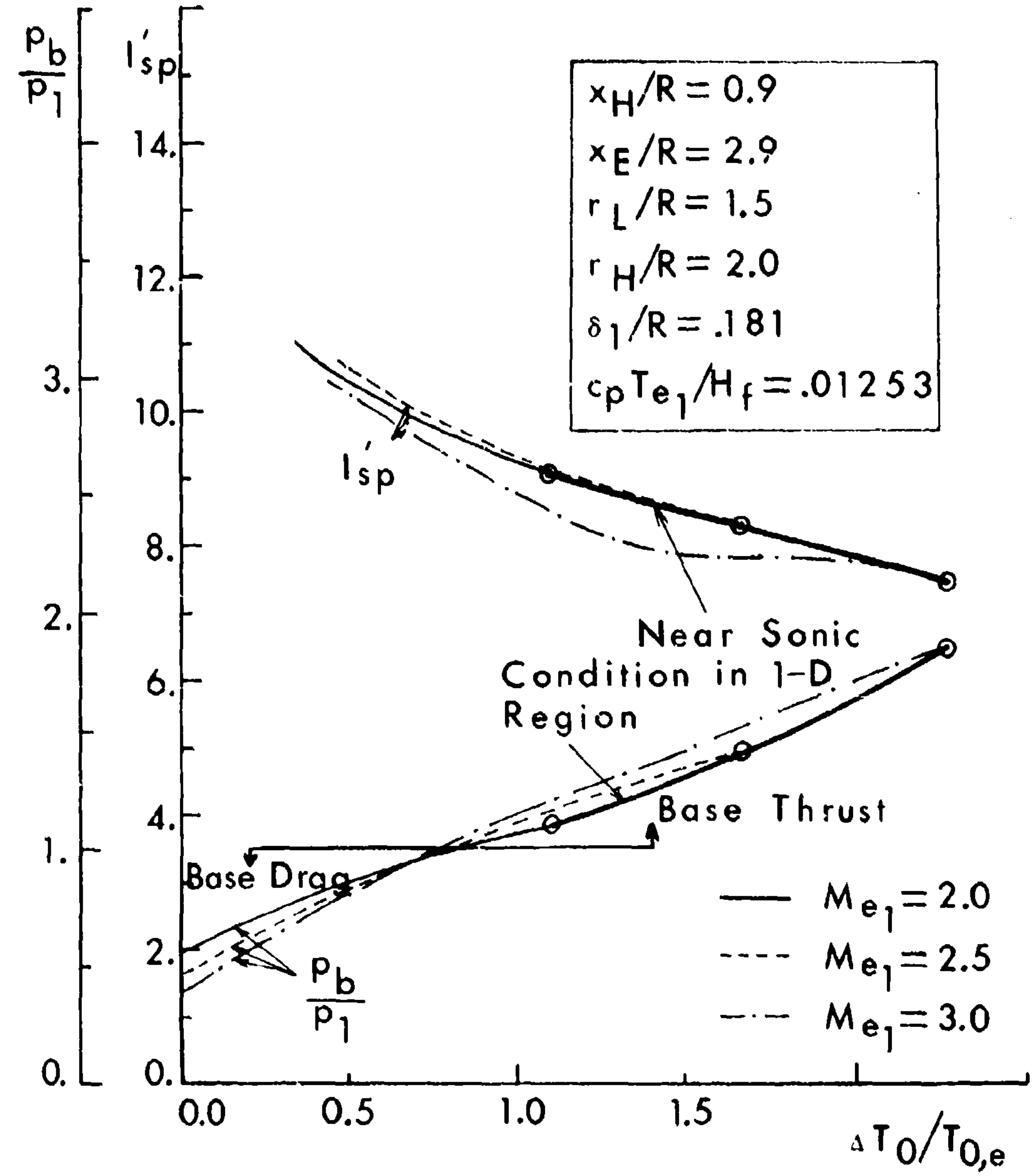


Figure 5. High Base Pressure Rise Runs: Specific Impulse and Base Pressure vs. Fuel-Air Ratio.
$I'_{sp} = I_{sp} M_{e1} g_o/c_{e1}$, $\Delta p_b' = \Delta p_b/(p_1 M_{e1})$

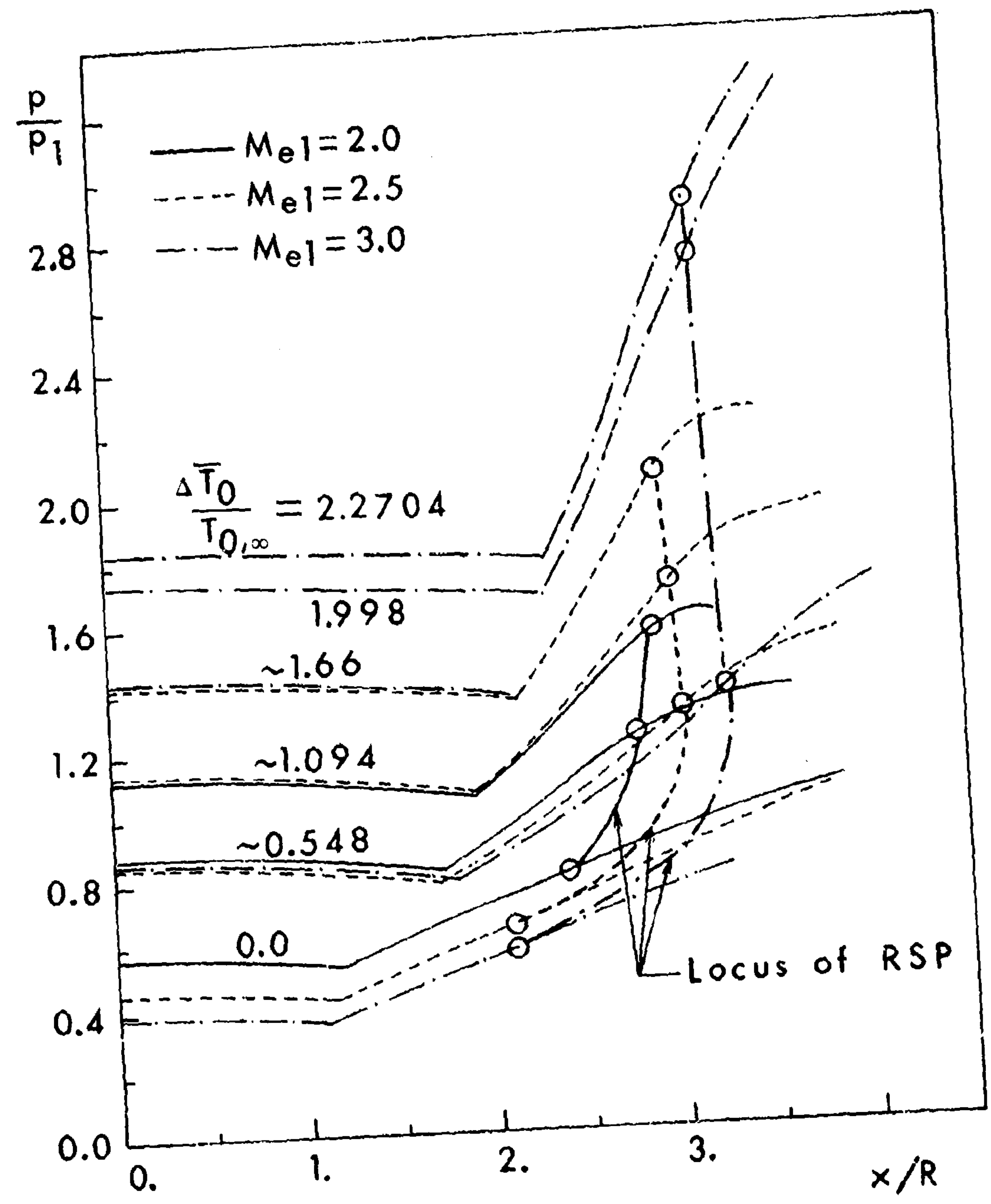


Figure 6. High Base Pressure Rise Runs: Centerline Pressure Variation with Axial Distance.

Next a parametric study was conducted to delineate the importance of various design variables. From dimensional analysis, the specific impulse can be written in terms of the non-dimensional parameters as follows

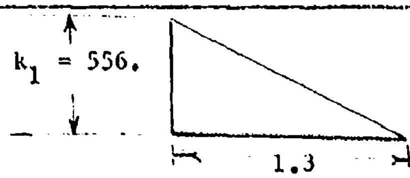
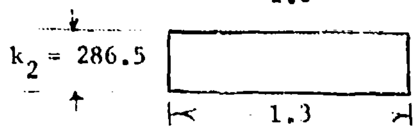
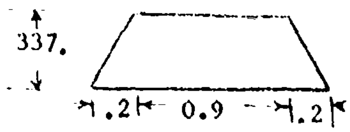
$$\frac{I_{sp}g_o}{c_{e1}} = f\left(\chi, M_{e1}, \frac{\delta_1}{R}, \frac{r_H}{R}, \frac{x_H}{R}, \frac{x_E - x_H}{R}, \frac{r_L}{R}, \frac{\Delta\bar{T}_o}{T_{o,\infty}}, \frac{c_p T_\infty}{H_f}\right)$$

To conduct this parametric study, the reference or design condition is chosen on the basis that, firstly, it should allow a study of a large variation of parameters, and secondly, the computer time for this study should be low. For a high heat addition and a high Mach number design point, the gradients occurring in the flow field are large, and a smaller step size and hence more computer time is required. The present study is made at medium heat addition, uniform combustion, with the following values of the parameters:

$$M_{e1} = 2.0$$
$$x_H/R = 1.5$$
$$(x_E - x_H)/R = 2.8$$
$$r_L/R = 1.5$$
$$r_H/R = 1.9$$
$$\delta_1/R = 0.181$$
$$\Delta\bar{T}_o/T_{o,\infty} = 0.398$$
$$c_p T_\infty/H_f = 0.01253$$

Table 1 shows the effect of the heat distribution parameter. Three types of heat distributions were studied, and these correspond to (a) a combustion zone which decreases in strength with distance, i.e., $\frac{d\bar{T}_o}{d\bar{x}} = K_1\left(1 - \frac{\bar{x}}{x_E - x_H}\right)$, (b) a constant combustion rate throughout the heat addition zone, i.e., $\frac{d\bar{T}_o}{d\bar{x}} = K_2$, and (c) a more realistic type of combustion - a slow rise in the

Table 1. Effect of heat distribution parameter

Type	Representation	$\frac{I_{sp} M_{el} g_o}{c_{el}}$
a	 $k_1 = 556.$; 1.3	12.32
b	 $k_2 = 286.5$; 1.3	12.35
c	 337.; .2, 0.9, .2	12.45

intensity of combustion representing the ignition stage, followed by uniform combustion and then a tapering off stage. As shown in Table 1, there is virtually no effect of the distribution. However, it should be noted that this interpretation is correct only if the combustion zone is small, so that all the compression waves hit in the initial portion of the near wake. If the combustion zone has a large length, type (a) will be the best since there will be less wasted heat.

The effect of freestream Mach number on specific impulse and base pressure rise is shown in Fig. 7. $I_{sp} M_{el} g_o/c_{el}$ and $\Delta p_b/(p_1 M_{el})$ are plotted instead of $I_{sp} g_o/c_{el}$ and $\Delta p_b/p_1$ since they are less dependent on the Mach number. I_{sp} falls from 252 secs. to 110 secs. as M_{el} is increased from 1.5 to 3.5. The variation of some typical features of the near wake, x_{RSP} and p_{RSP}, are also plotted on the same figure. The variation of these is almost opposite to what is obtained without heat addition.

Figure 8 shows the variation of various variables with upstream boundary layer thickness. The effect of the heat addition, in general, is to nullify the

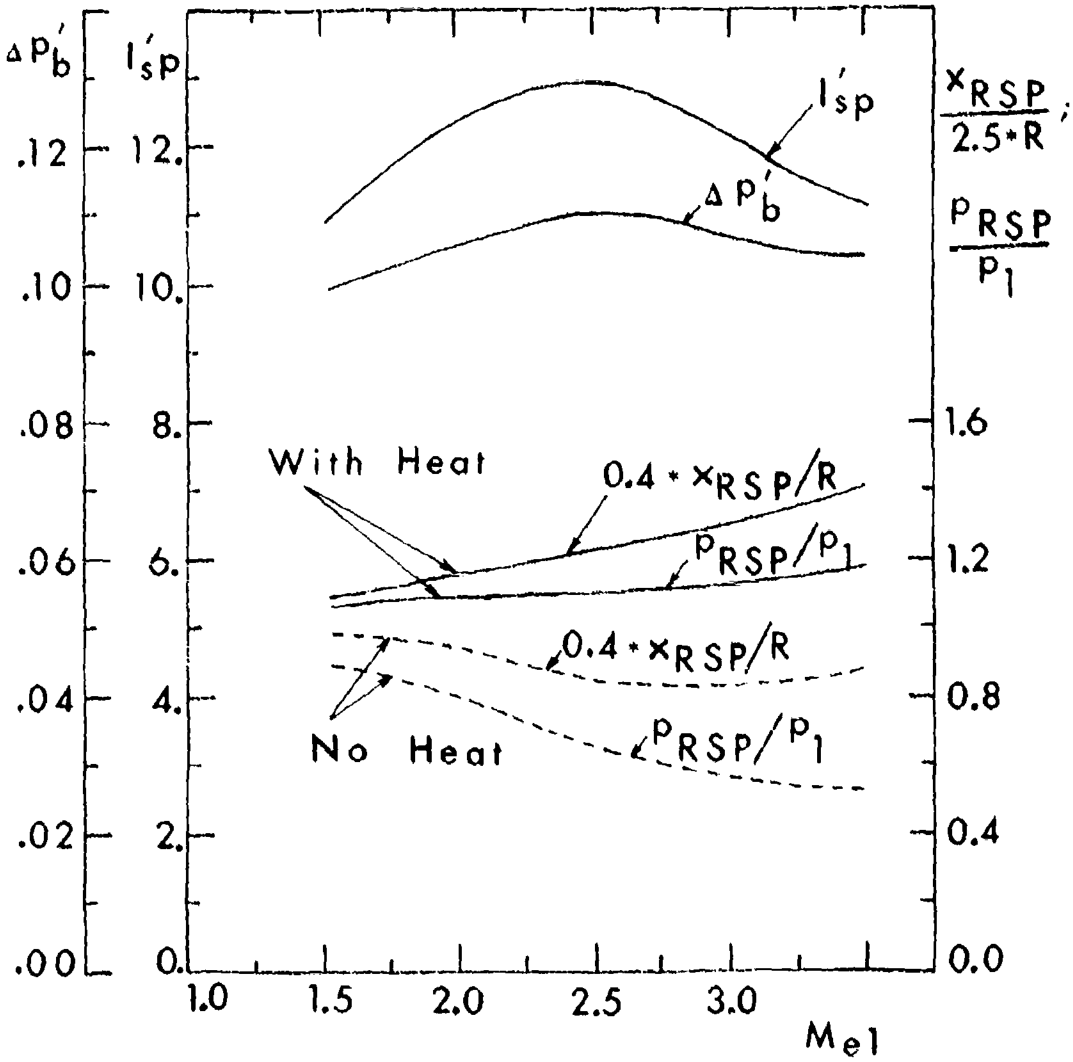


Figure 7. Effect of Freestream Mach Number.

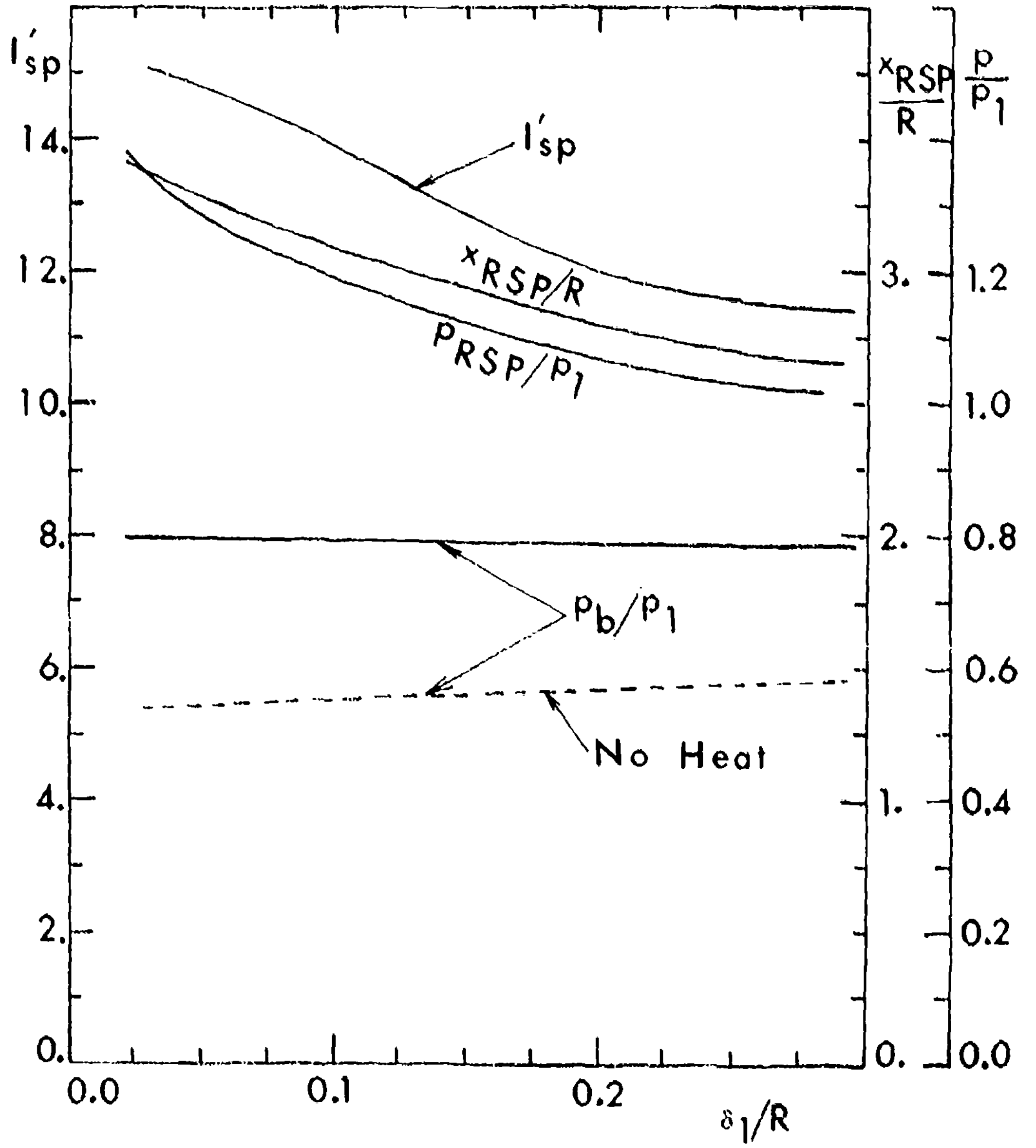


Figure 8. Effect of Upstream Boundary Layer Thickness.

effect of boundary layer thickness on base pressure without combustion. The specific impulse reaches an asymptotic value after a moderate thickness of the boundary layer.

The effect of variation of the combustible mass by varying the upper streamline position is shown in Fig. 9. The value of I_{sp} increases from 302 to 142 secs., and p_b increases from 0.638 to 0.935 as r_H/R is raised from 1.6 ($\frac{\dot{m}_{1-D}}{\rho_{c1} u_{c1} A_b} = 0.31$) to 2.4 ($\frac{\dot{m}_{1-D}}{\rho_{c1} u_{c1} A_b} = 3.51$). $\Delta p_{RSP}/\Delta p_b$, the shape parameter of the centerline pressure curve, almost remains constant (not shown), while x_{RSP}/R increases with an increase of r_H/R.

Figure 10 shows the variation of specific impulse parameter and pressure rise parameter with axial location of the heat addition zone. The flat maxima in these curves verify the results obtained with the variation of heat distribution parameter, χ. However, as the heat addition zone is moved farther and farther away from the base, part of the compression due to external burning affects the flow downstream of the wake critical point, and is ineffective in a base pressure alteration. Finally, a condition is reached when the presence of external burning is not felt at all by the near wake, and the same near wake details, as without combustion, are obtained. This figure also shows the variation of x_{RSP}/R and p_{RSP}/p_I. x_{RSP} varies linearly with x_H in most of the portion, as expected from the experiments with compression surfaces.

Figure 11 shows that there is an optimum rate of burning the fuel (or heat addition rate). The maximum is flat, as would be expected from the studies of variation of χ and x_H. The sharp fall in performance parameter after some stage is again due to the fact that part of the heat addition becomes useless. The characteristic properties of the centerline pressure distribution, x_{RSP}/R and $\Delta p_{RSP}/\Delta p_b$, have an almost similar shape.

The effect of the radial location of the heat addition zone on the performance, as shown in Fig. 12, is very weak. This is again due to the basic mechanism of raising the base pressure in an external burning system. The compression

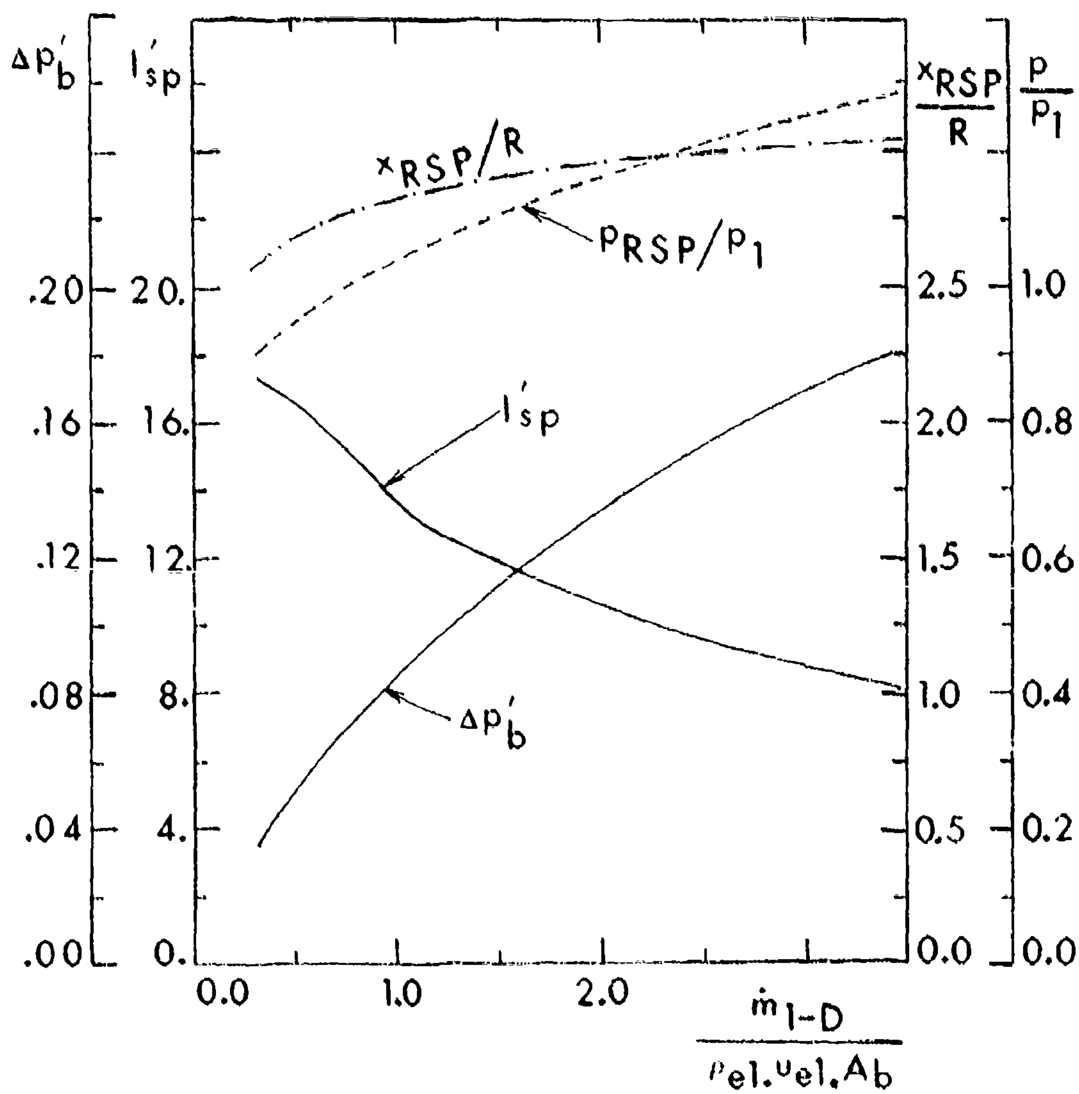


Figure 9. Effect of Combustible Mass by Varying r_{ll}/R.

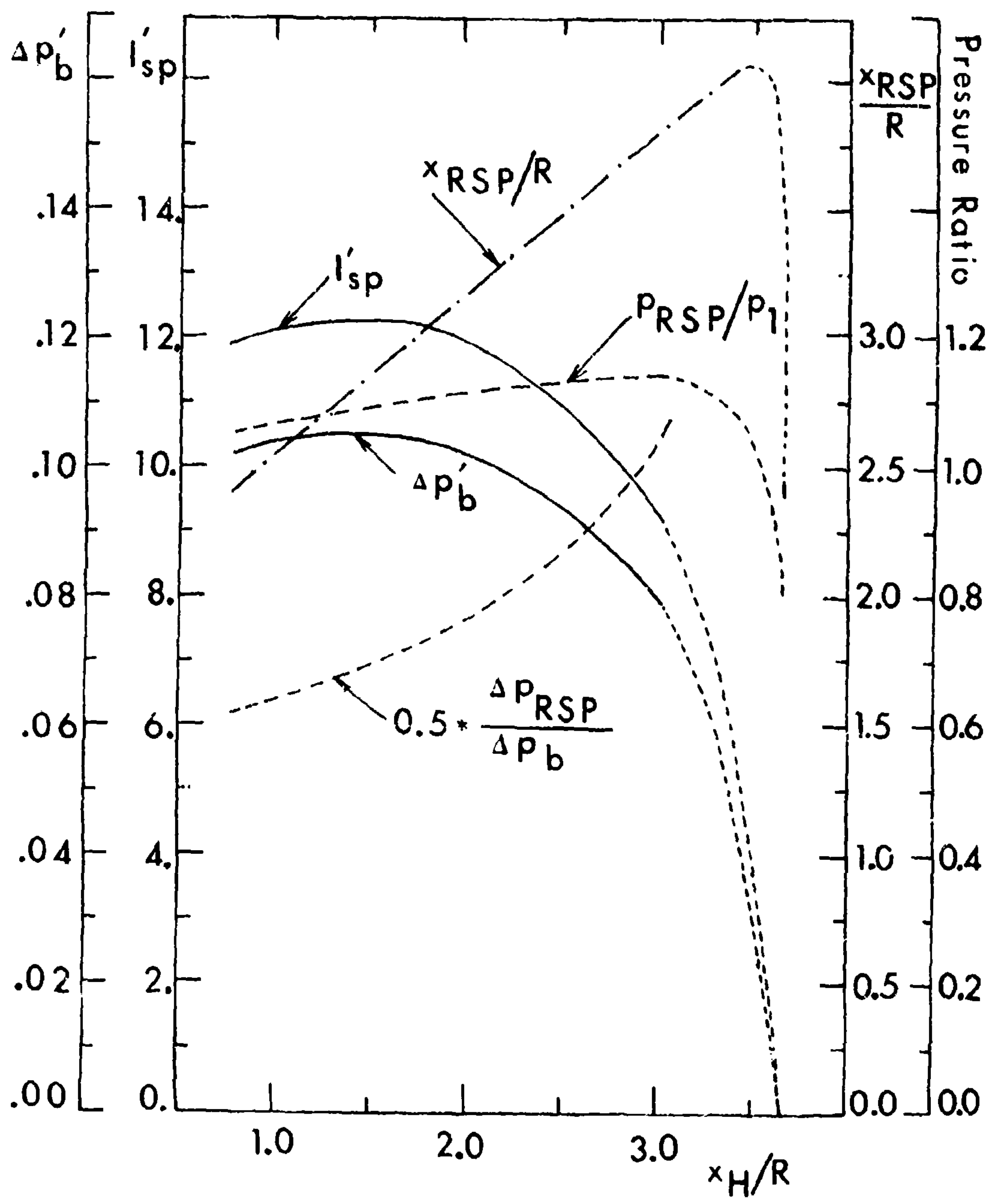


Figure 10. Effect of Axial Location of Combustion Zone.

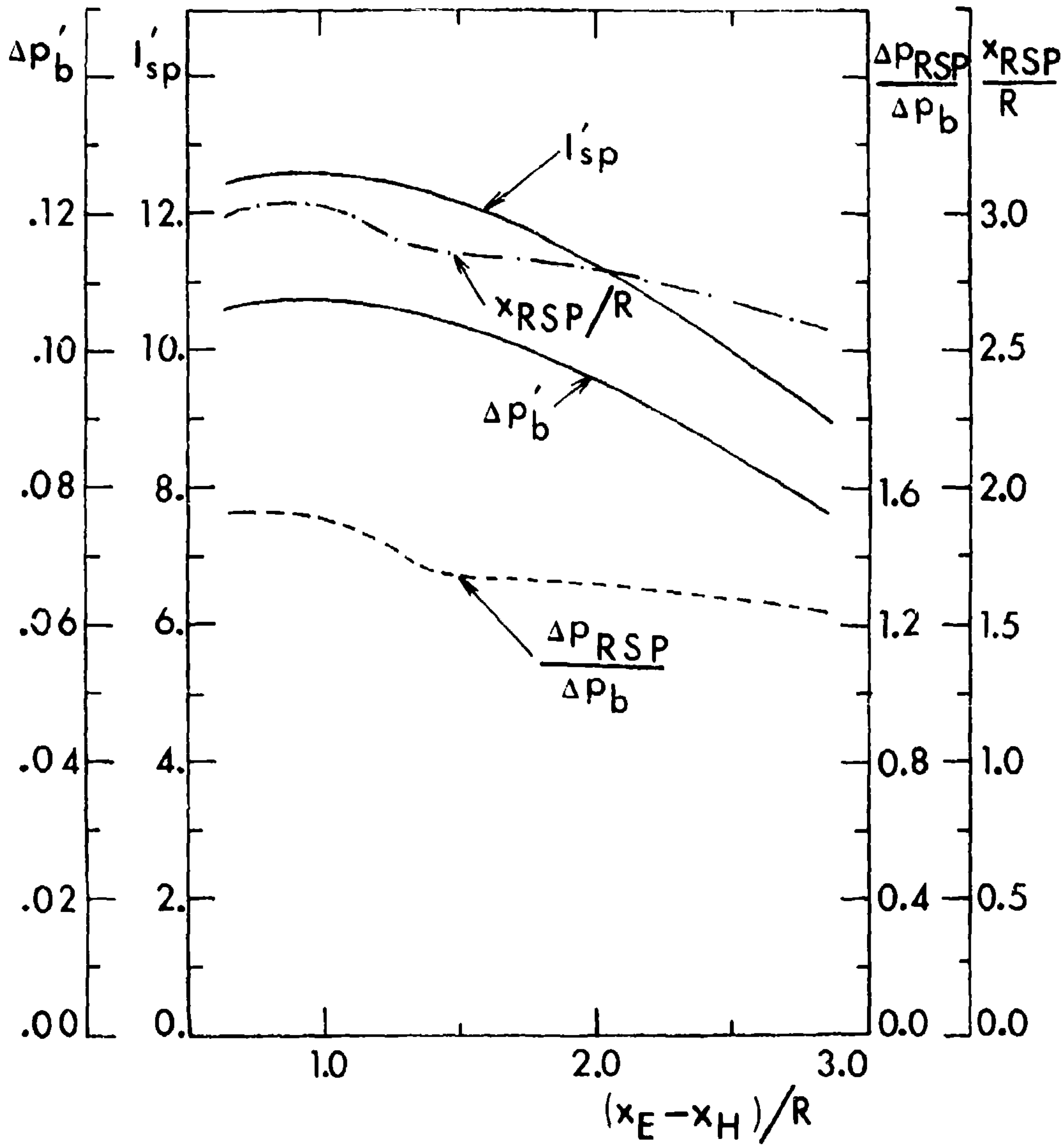


Figure 11. Effect of Rate of Burning.

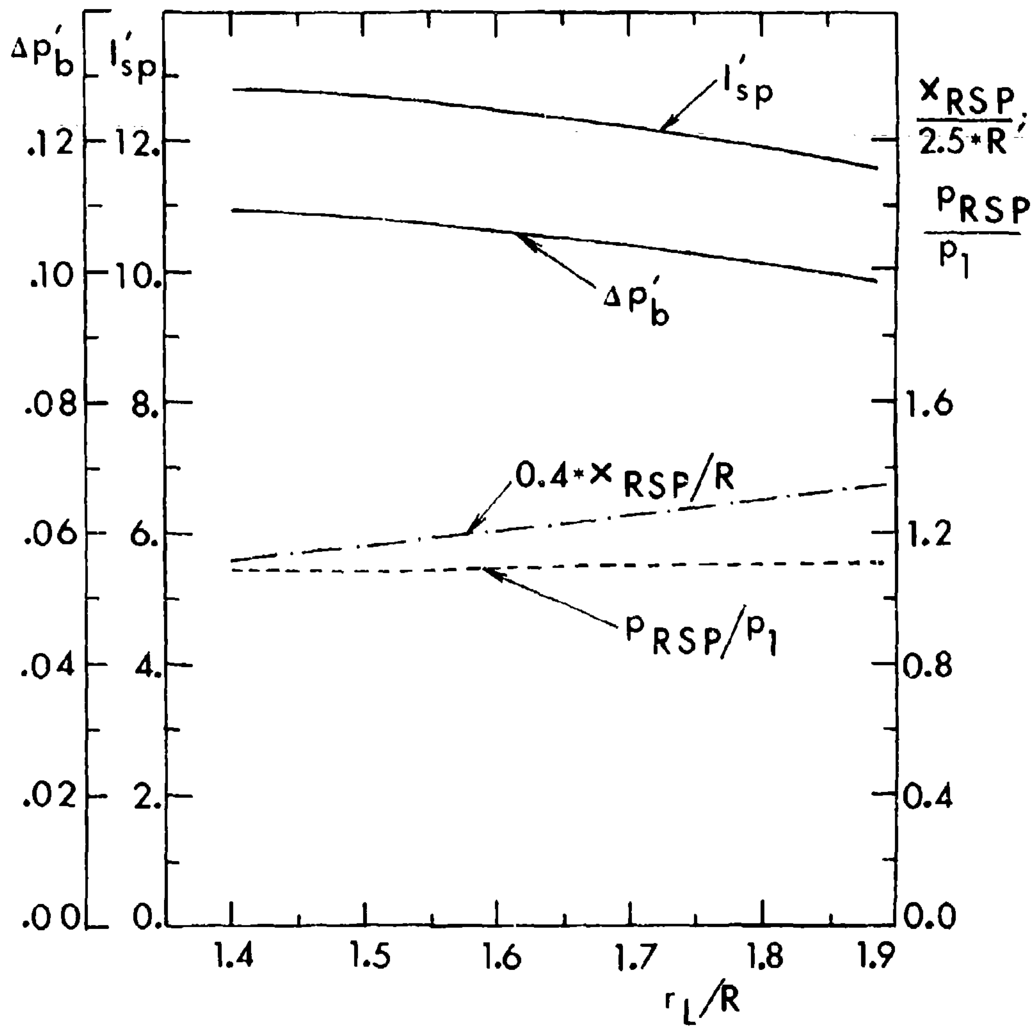


Figure 12. Effect of Radial Location of the Heat Addition Zone.

waves caused by the heat addition meet the viscous wake before the critical point for almost the whole range of r_L/R shown in the figure. x_{RSP} varies linearly with r_L (as would be expected from the study of variation of x_H), while p_{RSP} remains almost constant.

Figure 13 shows the effect of $\frac{\Delta\bar{T}_o}{T_{o,\infty}}$ $\left(\simeq \frac{F H_f}{c_p T_{o,\infty}}\right)$. Curves very similar to those presenting the effect of the change of combustible mass are obtained. I_{sp} varies from 250 to 189 secs. and p_b/p_1 varies from 0.613 to 1.011 as $\Delta\bar{T}_o/T_{o,\infty}$ changes from 0.07 to 0.94. The location of the RSP and $\Delta p_{RSP}/\Delta p_b$ approach asymptotic values with high values of $\Delta\bar{T}_o/T_{o,\infty}$.

Finally, the variation in performance with the freestream temperature is shown in Fig. 14. This variation, however, can be easily predicted. The base pressure is independent of T_∞ or Reynolds number when the flow is turbulent (except from the weak effect which comes through the boundary layer thickness). Using this information with the definition of I_{sp} as given by Eq. (18), it can be shown that $I_{sp} \propto \frac{1}{\sqrt{T_\infty}}$.

From the above study, the question of how small variations (intentional or unintentional) from the reference point will effect the performance can be answered. Some parameters like χ, x_H, (x_E-x_H) and r_L have previously been shown to have secondary effects on the performance for reasonable design conditions. Figure 15 is the plot of the specific impulse parameter against the total energy parameter, and is obtained from Figs. 9 and 13. It shows that for the previous reference conditions, it is more efficient to change the fuel-air ratio if a little higher base drag reduction is required, and to change r_H/R if a little lesser base drag reduction is required for the same projectile, same Mach number and altitude of flight. However, no optimization studies have been carried out.

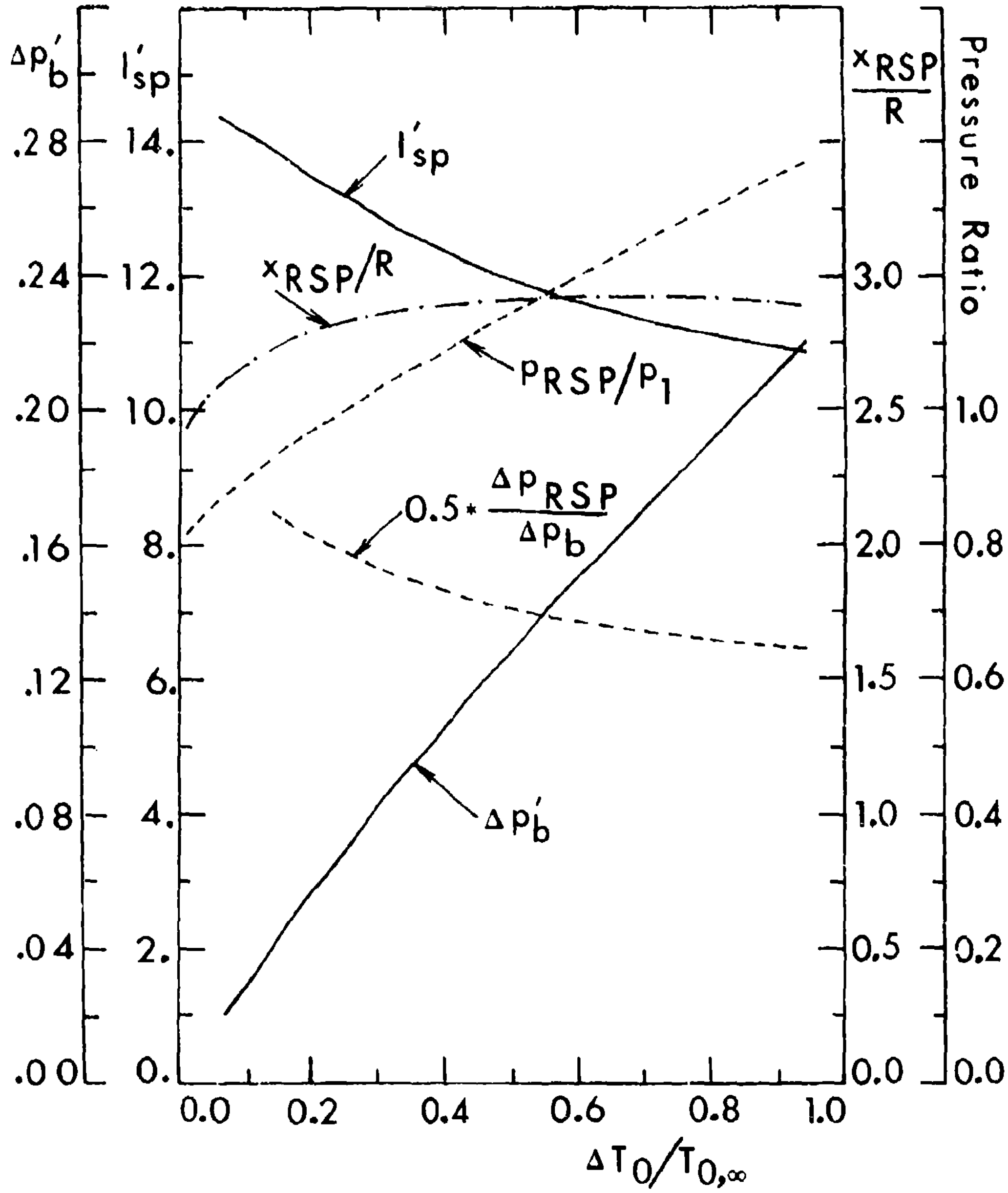


Figure 13. Effect of Fuel-Air Ratio.

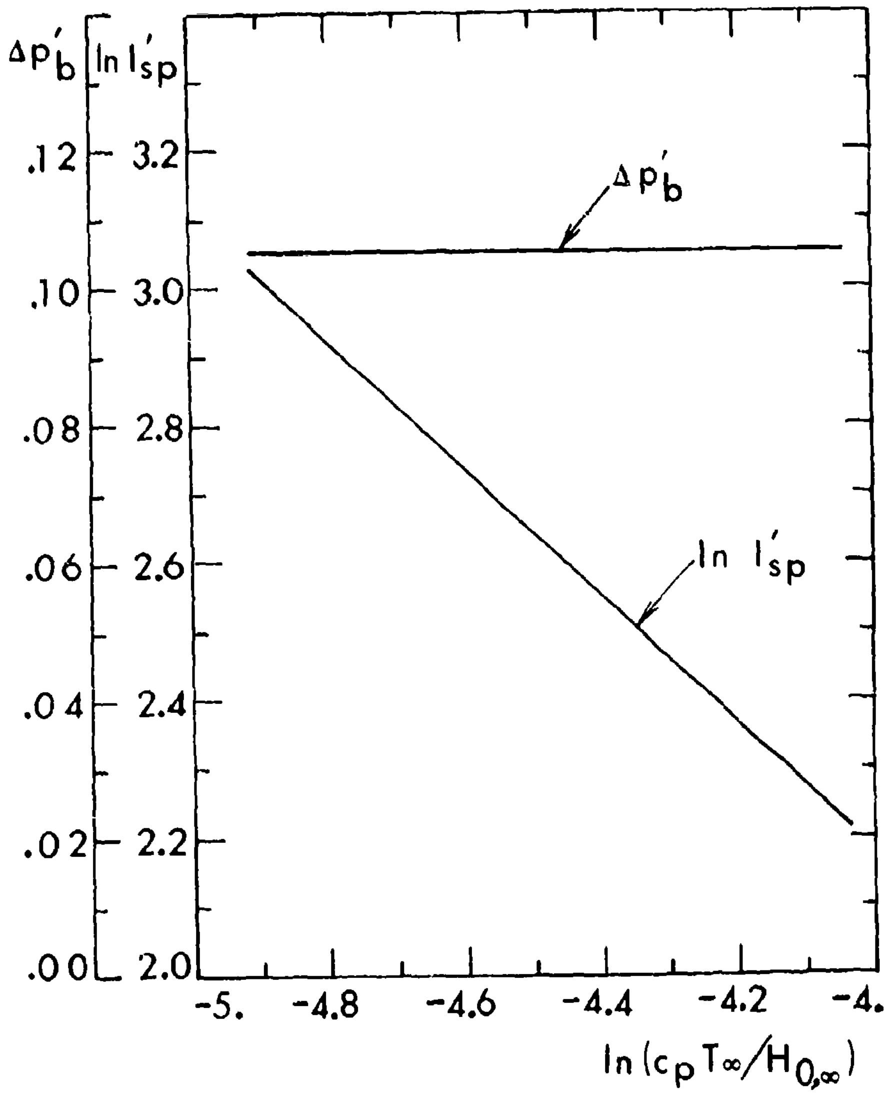


Figure 14. Effect of Freestream Temperature.

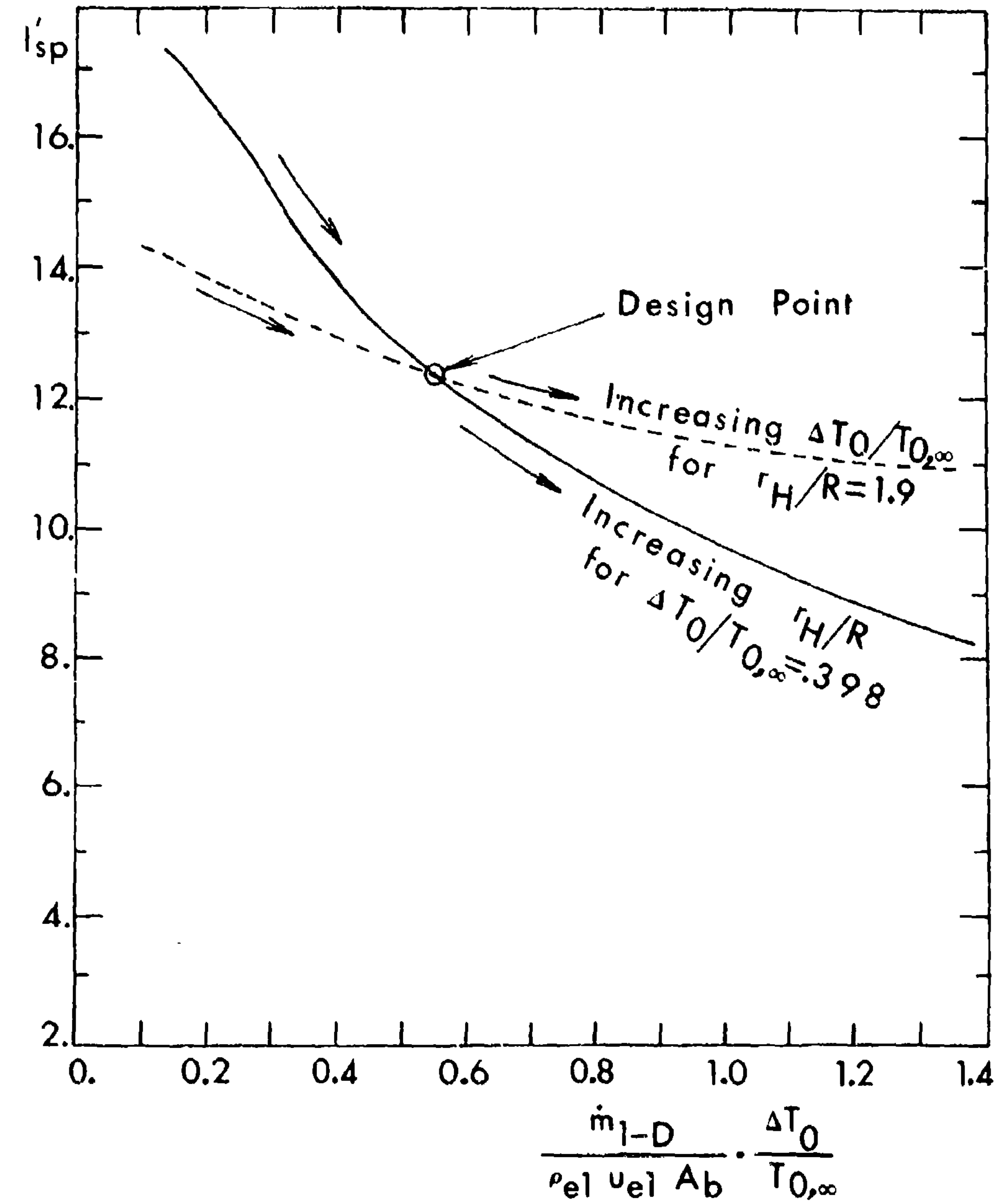


Figure 15. Effect of Perturbations on Performance.

V. Conclusions and Comments

The present study shows that the performance of an external propulsion system is strongly dependent only on the upstream Mach number, fuel-air ratio, the combustible mass and the fuel calorific value. Also the system becomes more efficient with a cold freestream. For preliminary design of this system, reasonable values of heat distribution parameter, axial and radial locations, and length of the heat addition zone can be assumed, as their effect on the performance is only secondary. Thus for a given projectile flying at a given Mach number and altitude, and given base pressure rise, the optimization problem is reduced to the selection of r_H/R and $\Delta T_o/T_{o,\infty}$. One rule of thumb which came out of these studies, and should be helpful in extrapolating experimental results, is that, except for very low heat additions, base pressure remains constant with Mach number and upstream boundary layer thickness for the same values of other parameters.

The question whether the present values indicate an upper limit of performance is unanswered. To answer this question, one has to consider two effects, viz., the losses due to viscosity and the mass entrainment in the heat addition zone due to mixing. An improvement in modelling of the external burning zone is required to answer this question satisfactorily.

Finally, the present calculations show that although high net thrust can be obtained using the external burning method, this method is not efficient unless a high Btu fuel is used. Its performance falls short of that of conventional rocket propulsion systems. However, it is still attractive because of the simplicity of design. Its performance probably can be boosted by combining this method with the base burning method, which is very efficient for low base pressure rise values. More experimental and theoretical work are warranted to check these conclusions.

Acknowledgement

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Appendix

The present theory was compared with experiments[5] in which external compression was carried out by modifying the shape of the wind tunnel test section. The conditions on the initial Mach wave at the corner were obtained using potential flow theory, ignoring shocks and the boundary layer thickness. The effect of the boundary layer in these calculations was taken into account in a similar fashion as in the original theory. The rotational layer was ignored for simplicity, and, also, since the base pressures were high with compression surfaces, the viscous forces remain predominant in whole of the initial boundary layer. Figures 16 and 17 show both the experimental and theoretical results. The agreement between theory and experiment is reasonable, although the base pressure obtained is about 15% less than given by experiment. Some of the difference may be due to (i) the approximations made in calculating the conditions on the initial Mach wave, (ii) the slight compression caused by the boundary layer on the tunnel wall, and (iii) the slightly improper accounting of the boundary layer thickness effect.

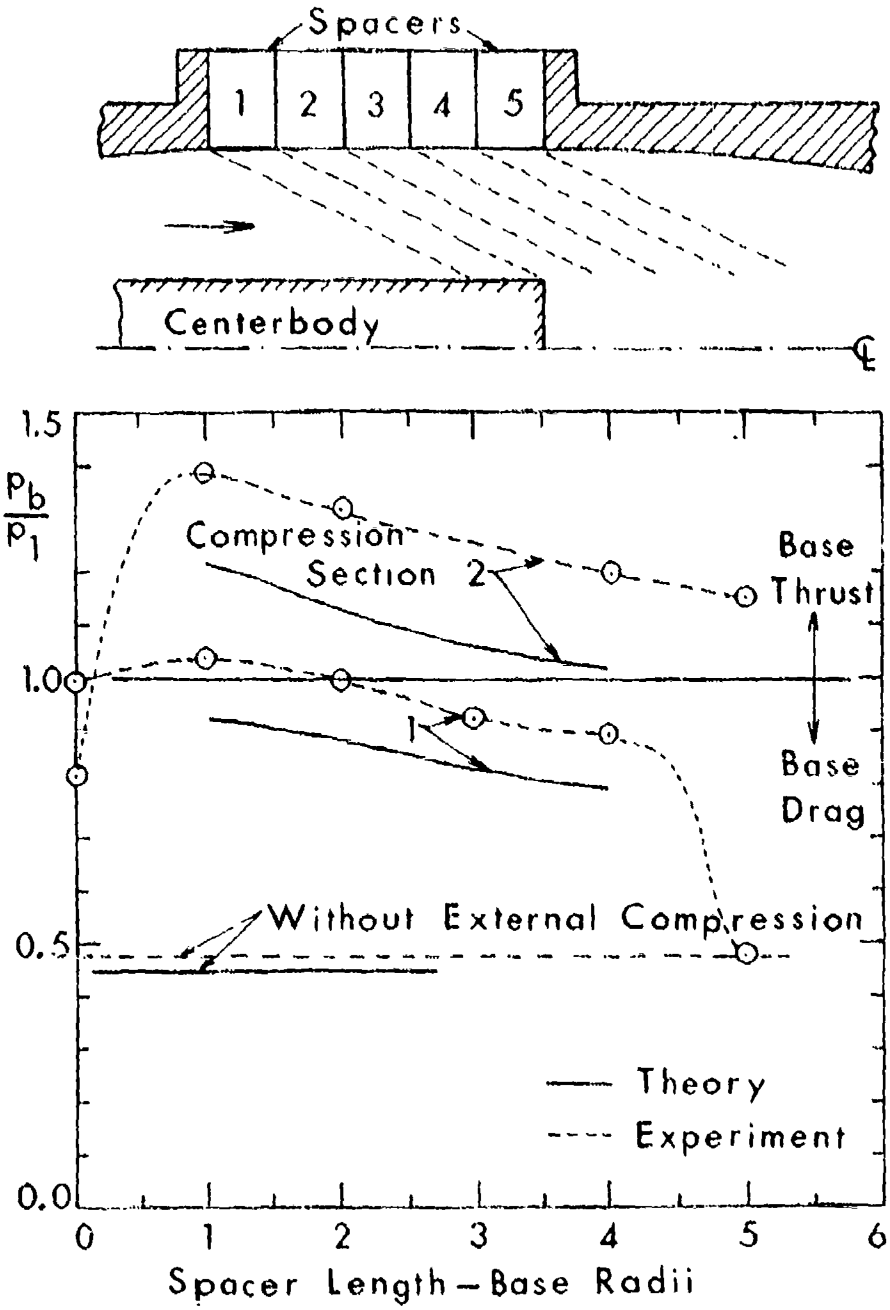

Figure 16. Computations with External Compression: Effect of External Compression Strength and Location on Base Pressure.

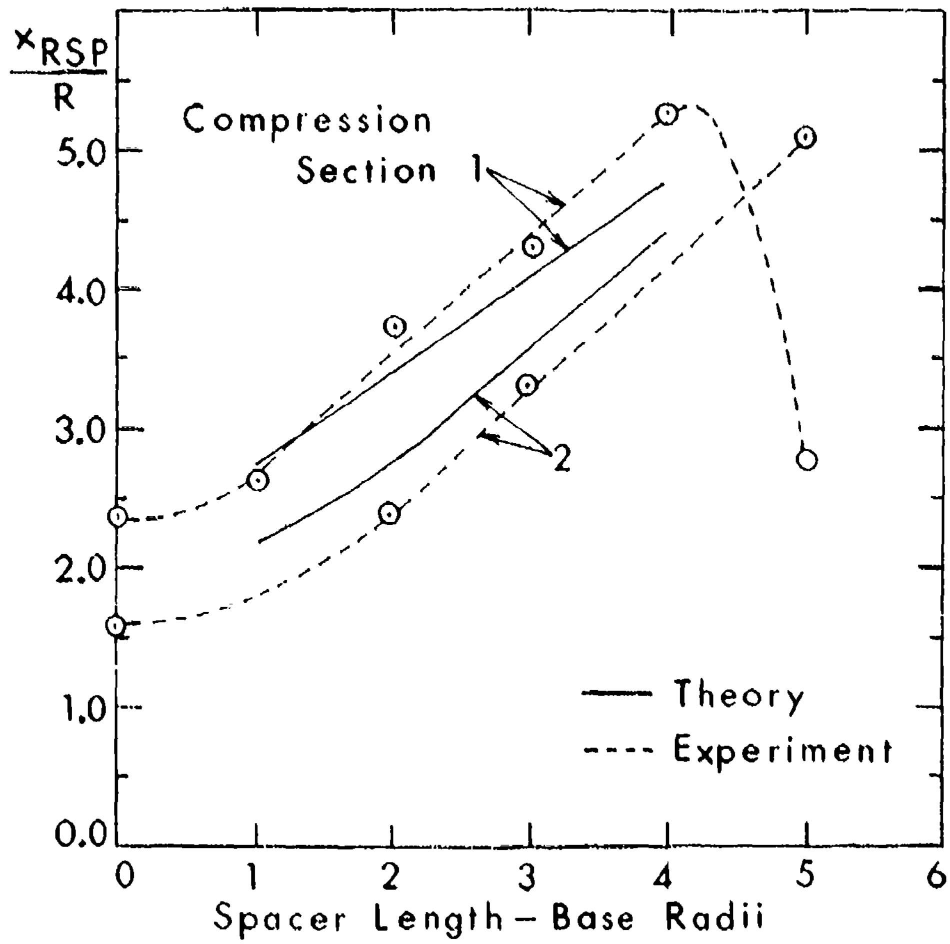

Figure 17. Computations with External Compression: Effect of External Compression Strength and Location on Location of the RSP.